LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION - **STATISTICS**

FIRST SEMESTER – NOVEMBER 2011

ST 1815/1810 - ADVANCED DISTRIBUTION THEORY

Date : 03-11-2011 Dept. No. Max. : 100 Marks

Time : 1:00 - 4:00

SECTION - A

Answer ALL questions. Each carries TWO marks. (10 x 2 = 20 marks)

1. Define distribution function of a random variable and state its properties.
2. Find the mean and variance of discrete uniform distribution.
3. Let X be exponential with parameter θ. Obtain the p.d.f. of truncated exponential with support (2 , ∞) .
4. Let X_1 and X_2 have iid Poisson distribution P(λ). Find the conditional distribution of $X_1 \mid X_1 + X_2 = n$ at x.
5. Show that the marginals of a bivariate discrete uniform need not be discrete uniform.
6. Let $(X_1, X_2) \sim BVP(\lambda_1, \lambda_2, \lambda_{12})$. Find $V(X_1 \mid X_2 = x_2)$.
7. Let $X \sim \Lambda(\mu, \sigma^2)$. Show that $\frac{1}{X} \sim \Lambda(-\mu, \sigma^2)$.
8. Show that exponential distribution satisfies lack of memory property.
9. Let $(X_1, X_2) \sim BVE(\lambda_1, \lambda_2, \lambda_{12})$. Find the distribution of $X_1 \wedge X_2$.
10. Define compound distribution and write the formula to obtain its p.d.f. when (i) θ is discrete (ii) θ is continuous.

SECTION - B

Answer any FIVE questions. Each carries EIGHT marks. (5 x 8 = 40 marks)

11. Let the distribution function of X be

$$F(x) = \begin{cases} 0, & x < -1 \\ \frac{(x+2)}{4}, & -1 \le x < 1 \\ 1, & 1 \le x < \infty \end{cases}$$

Find (i) the decomposition of F, (ii) MGF of X.

12. Let $\{ X_n, n \ge 1 \}$ be a sequence of random variables such that X_n is discrete uniform on $\{ \frac{1}{n}, \frac{2}{n}, \ldots, \frac{n}{n} \}$, n = 1,2,3,…..Show that $X_n \xrightarrow{d} X$ where X is U (0,1).
13. State and prove a characterization of Bernoulli Distribution through moments.
14. Verify that Binomial , Poisson and Log-series distributions are Power –series distributions.
15. Let $(X_1, X_2) \sim BB(n, p_1, p_2, p_{12})$. Write the two regression equations and hence obtain the correlation coefficient between X_1 and X_2.
16. Let $(X_1, X_2) \sim BB(n, p_1, p_2, p_{12})$. Stating the conditions , show that (X_1, X_2) tends to $BVP(\lambda_1, \lambda_2, \lambda_{12})$.
17. Let $(X_1, X_2) \sim BVN(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$. Find the marginal distributions of X_1 and X_2.
18. Define non-central 't' distribution and derive its p.d.f.

SECTION – C

Answer any TWO questions. Each carries TWENTY marks. (2 x 20 = 40 marks)

19(a) State and prove a characterization of geometric distribution through order statistics. (16)

(b) Let X_1, X_2, X_3 be independent normal variates such that $E(X_1) = 1$, $E(X_2) = 3$, $E(X_3) = 2$ and $V(X_1) = 2$, $V(X_2) = 2$, $V(X_3) = 3$. Examine the independence of $X_1 + X_2$ and $X_1 - X_2$. (4)

20(a) State and prove a characterization of Normal distribution. (12)

(b) Define log-normal distribution and find its p.d.f., mean, and variance. (8)

21(a) Let $X_1 \sim G(\alpha, p_1)$, $X_2 \sim G(\alpha, p_2)$ and X_1 is independent of X_2.

Prove that

(i) $X_1 + X_2 \sim G(\alpha, p_1 + p_2)$,

(ii) $X_1 / (X_1 + X_2)$ ~ Beta distribution of first kind,

(iii) $X_1 + X_2$ is independent of $X_1 / (X_1 + X_2)$. (16)

(b) State and prove additive property of Bivariate Binomial distribution. (4)

22(a) Define non-central Chi-square distribution and derive its MGF. (18)

(b) Define Quadratic form in normal variables. (2)

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